PROBLEM SOLVING WORKSHOP
LESSON 7.6

Using ALTERNATIVE METHODS

Another Way to Solve Examples 2 and 7, pp. 516 and 519

MULTIPLE REPRESENTATIONS In Examples 2 and 7 on pages 516 and 519, respectively, you solved exponential and logarithmic equations algebraically. You can also solve such equations using tables and graphs.

PROBLEM 1

Solve the following exponential equation: $4^x = 11$.

METHOD 1

Using a Table One way to solve the equation is to make a table of values.

STEP 1 **Enter** the function $y = 4^x$ into a graphing calculator.

```
Y1=4^X
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

STEP 2 **Create** a table of values for the function.

X	Y1
1.5	8
1.6	9.1896
1.7	10.556
1.8	12.126
1.9	13.929

X=1.7

STEP 3 **Scroll** through the table to find when $y = 11$. The table in Step 2 shows that $y = 11$ between $x = 1.7$ and $x = 1.8$.

▶ The solution of $4^x = 11$ is between 1.7 and 1.8.

METHOD 2

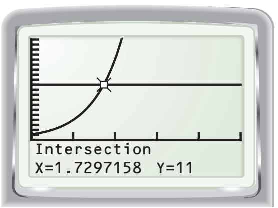
Using a Graph You can also use a graph to solve the equation.

STEP 1 **Enter** the functions $y = 4^x$ and $y = 11$ into a graphing calculator.

```
Y1=4^X
Y2=11
Y3=
Y4=
Y5=
Y6=
Y7=
```

STEP 2 **Graph** the functions. Use the *intersect* feature to find the intersection point of the graphs. The graphs intersect at about (1.73, 11).

Use a viewing window of $0 \le x \le 5$ and $0 \le y \le 20$.



▶ The solution of $4^x = 11$ is about 1.73.

Alternative Strategy

Method 1 Example 2 on page 516 can be solved by using a table. This method allows students to see how the value of 4^x gets closer and closer to 11 as the value of x approaches the solution of the equation. This allows students to approach solving the equation entirely in terms of exponents, rather than working with logarithms, as in the algebraic solution in Lesson 7.6.

Method 2 Example 2 on page 516 can also be solved by using a graph. This method allows students to visualize the solution as the x-coordinate of the intersection point of the two functions $y_1 = 4^x$ and $y_2 = 11$. Again, the emphasis is on the exponential function, and there is no need to use logarithms.

Avoiding Common Errors

Method 1 Some students may have trouble approximating the solution because they are looking at the wrong portion of the table. To avoid this, ask students to estimate the value by mental math before working with their calculators. Since $4^1 = 4$ and $4^2 = 16$, they should immediately see that they should concentrate on values of x between $x = 1$ and $x = 2$.

Graphing Calculator

Method 1 Students may feel that "between 1.7 and 1.8" is not a really a solution, and notice that the table displayed on this page shows y-values of 10.556 and 12.126, which are not very close to 11. Ask them how they might change the table to get a more accurate solution. They should see that to get another decimal place of accuracy, they will need to change ΔTbl to 0.01 and look between $x = 1.71$ and $x = 1.80$. The y-value at $x = 1.73$ is 11.004, giving 1.73 as a solution that is a good approximation to the actual irrational solution.

Alternative Strategy

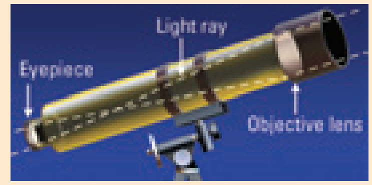
Method 1 Example 7 on page 519 can be solved by using a table. This method allows students to see how the value of M increases as the value of D increases in the application and does not require the series of steps used in the algebraic solution given in Lesson 7.6. The table works well in this case because the answer is exact.

Teaching Strategy

It can take a long time to scroll through the table up to $x = 100$ if you start at $x = 1$ and use ΔTbl $= 1$, as shown in Step 2. Ask students for suggestions on how to do this more efficiently. Good suggestions would be to estimate the solution mentally before using the table and starting with a larger ΔTbl value, such as ΔTbl $= 10$.

PROBLEM 2

ASTRONOMY The *apparent magnitude* of a star is a measure of the brightness of the star as it appears to observers on Earth. The apparent magnitude M of the dimmest star that can be seen with a telescope is given by the function



$$M = 5 \log D + 2$$

where D is the diameter (in millimeters) of the telescope's objective lens. If a telescope can reveal stars with a magnitude of 12, what is the diameter of its objective lens?

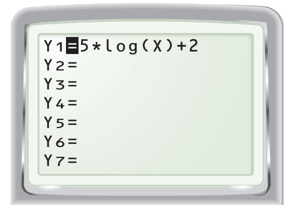
METHOD 1

Using a Table Notice that the problem requires solving the following logarithmic equation:

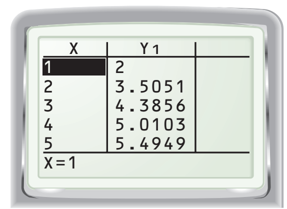
$$5 \log D + 2 = 12$$

One way to solve this equation is to make a table of values. You can use a graphing calculator to make the table.

STEP 1 **Enter** the function $y = 5 \log x + 2$ into a graphing calculator.

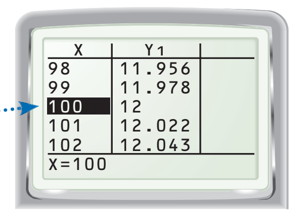


STEP 2 **Create** a table of values for the function. Make sure that the x-values are in the domain of the function ($x > 0$).



STEP 3 **Scroll** through the table of values to find when $y = 12$.

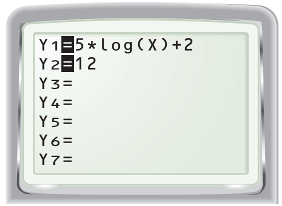
The table shows that $y = 12$ when $x = 100$.



▶ To reveal stars with a magnitude of 12, a telescope must have an objective lens with a diameter of 100 millimeters.

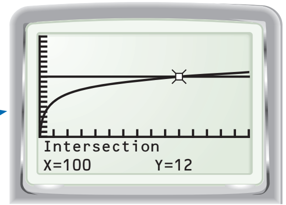
METHOD 2 **Using a Graph** You can also use a graph to solve the equation $5 \log D + 2 = 12$.

STEP 1 **Enter** the functions $y = 5 \log x + 2$ and $y = 12$ into a graphing calculator.



STEP 2 **Graph** the functions. Use the *intersect* feature to find the intersection point of the graphs. The graphs intersect at (100, 12).

Use a viewing window of $0 \le x \le 150$ and $0 \le y \le 20$.



▸ To reveal stars with a magnitude of 12, a telescope must have an objective lens with a diameter of 100 millimeters.

PRACTICE

EXPONENTIAL EQUATIONS **Solve the equation using a table and using a graph.** **1–4. Sample viewing windows are given.**

1. $8 - 2e^{3x} = -14$ **about 0.799 (viewing window: $0 \le x \le 2$, $-16 \le y \le 6$)**
2. $7 - 10^{5-x} = -9$ **about 3.80 (viewing window: $0 \le x \le 10$, $-16 \le y \le 6$)**
3. $e^{5x-8} + 3 = 15$ **about 2.10 (viewing window: $0 \le x \le 5$, $-1 \le y \le 17$)**
4. $1.6(3)^{-4x} + 5.6 = 6$ **about 0.315 (viewing window: $0 \le x \le 5$, $-1 \le y \le 12$)**

LOGARITHMIC EQUATIONS **Solve the equation using a table and using a graph.** **5–8. Sample viewing windows are given.**

5. $\log_2 5x = 2$ **0.8 (viewing window: $0 \le x \le 5$, $-3 \le y \le 5$)**
6. $\log(-3x + 7) = 1$ **-1 (viewing window: $-5 \le x \le 5$, $-1 \le y \le 2$)**
7. $4 \ln x + 6 = 12$ **about 4.48 (viewing window: $0 \le x \le 6$, $-5 \le y \le 14$)**
8. $11 \log(x + 9) - 5 = 8$ **about 6.20 (viewing window: $0 \le x \le 10$, $-1 \le y \le 10$)**
9. **ECONOMICS** From 1998 to 2003, the United States gross national product y (in billions of dollars) can be modeled by $y = 8882(1.04)^x$ where x is the number of years since 1998. Use a table and a graph to find the year when the gross national product was \$10 trillion. **2001**
10. **WRITING** In Method 1 of Problem 1 on page 523, explain how you could use a table to find the solution of $4^x = 11$ more precisely. **See margin.**
11. **WHAT IF?** In Problem 2 on page 524, suppose the telescope can reveal stars of magnitude 14. Find the diameter of the telescope's objective lens using a table and using a graph. **about 251.19 mm**
12. **FINANCE** You deposit \$5000 in an account that pays 3% annual interest compounded quarterly. How long will it take for the balance to reach \$6000? Solve the problem using a table and using a graph. **6.25 yr**
13. **OCEANOGRAPHY** The density d (in grams per cubic centimeter) of seawater with a salinity of 30 parts per thousand is related to the water temperature T (in degrees Celsius) by the following equation:

$$d = 1.0245 - e^{0.1226T - 7.828}$$

For deep water in the South Atlantic Ocean off Antarctica, $d = 1.0241$ g/cm^3. Use a table and a graph to find the water's temperature. **about 0.03225°C**

Alternative Strategy

Method 2 Example 7 on page 519 can also be solved by using a graph. This method allows students to visualize how the value of *M* increases as the value of *D* increases in the application. It shows students how a logarithmic graph first increases rapidly and then at a slower and slower rate.

10. ***Sample answer:*** **Once you find a range of values, keep adjusting ΔTbl on the calculator to find values to a smaller decimal place.**